Exercise (6) Ch. 2

هندسة إحداثية

1. Form the eq. that represent the following pair of lines:

i. $2x+y=0$ & $3x-2y=0$

Ans.

$(2x+y)(3x-2y)=0 \Rightarrow \boxed{6x^2-xy-2y^2=0}$ ※

ii. $2x+y-1=0$ & $x-4y+1=0$

Ans.

$(2x+y-1)(x-4y+1)=0 \Rightarrow$

$2x^2-8xy+2x+xy-4y^2+y-x+4y-1=0$, Then

$\boxed{2x^2-7xy-4y^2+x+5y-1=0}$ ※

4. Find the value of λ such that the two lines $3x^2-8xy+\lambda y^2=0$ are perpendicular.

Ans. for the pair of lines $ax^2+2hxy+by^2=0$, they are perpendicular when $a+b=0$. hence

$3+\lambda=0 \Rightarrow \boxed{\lambda=-3}$

5. Find the condition that one of the lines $ax^2+2hxy+by^2=0$ may coincides with one of the lines $Ax^2+2Hxy+By^2=0$

Ans: see example (5) P 42

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// // (6)

// // (7)

// // (8)

2. Show that each of the ...

find the angle betⁿ them. revise ex(2) P37

{ بقية مسائل هذا السؤال بنفس الطريقة. }

(iii) $x^2+8xy+y^2+16x+4y+4=0$

Ans: $\underset{\text{معامل } x^2}{a=1}$, $\underset{\text{نصف معامل } xy}{h=4}$, $\underset{\text{معامل } y^2}{b=1}$, $\underset{\text{نصف معامل } x}{g=8}$, $\underset{\text{نصف معامل } y}{f=2}$, $\underset{\text{الحد المطلق}}{C=4}$, Then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & 4 & 8 \\ 4 & 1 & 2 \\ 8 & 2 & 4 \end{vmatrix} = 1(4-4)-4(16-16)+8(8-8)=0 \quad \text{, Then}$$

$x^2+8xy+y^2+16x+4y+4=0$ is eq.n of Pair of lines.

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} \Rightarrow \tan\theta = \frac{2\sqrt{16-1}}{2} \Rightarrow \boxed{\theta = 75^\circ\ 31'\ 20.96''}$$

3. Find λ such that the following eq.ns represent pair of lines:

i. $12x^2-10xy+2y^2+11x-5y+\lambda=0$

Ans: $a=12$, $b=2$, $h=-5$, $g=\frac{11}{2}$, $f=-5/2$, $C=\lambda$, Then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 12 & -5 & \frac{11}{2} \\ -5 & 2 & -5/2 \\ \frac{11}{2} & -\frac{5}{2} & \lambda \end{vmatrix} = 0$$

باقي مسائل هذا السؤال له نفس الفكرة

$$12\left(2\lambda-\frac{25}{4}\right)+5\left(-5\lambda+\frac{55}{4}\right)+\frac{11}{2}\left(\frac{25}{2}-11\right)=0$$

$$24\lambda-75-25\lambda+\frac{275}{4}+\frac{275}{4}-\frac{121}{2}=0 \Rightarrow \boxed{\lambda=2} \#$$

ii. $x^2+2\lambda xy+y^2+6x+2y+9=0$

Ans. $a=1$, $h=\lambda$, $b=1$, $g=3$, $f=1$, $C=9$, Then

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & \lambda & 3 \\ \lambda & 1 & 1 \\ 3 & 1 & 9 \end{vmatrix} = 0$$

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$$1(9-1)-\lambda(9\lambda-3)+3(\lambda-3)=0$$

$$8-9\lambda^2+3\lambda+3\lambda-9=0$$

$$9\lambda^2-6\lambda+1=0 \Rightarrow (3\lambda-1)^2=0 \Rightarrow (3\lambda-1)=0 \Rightarrow \boxed{\lambda=\frac{1}{3}} \#$$

(انظر الخلف)

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Find the bisectors eq.n for

ii. $3x^2+8xy+4y^2=0$

Ans. The eq.n is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h} \Rightarrow \frac{x^2-y^2}{3-4}=\frac{xy}{4}$$

$$4x^2-4y^2=-xy \Rightarrow \boxed{4x^2+xy-4y^2=0}$$

ii. $x^2+xy-6y^2+7x+31y-18=0$

Ans. transform the eq.n to Point (h,K) to eleminate 1st degree terms

put $x=x'+h$ and $y=y'+K$, Then

$$(x'+h)^2+(x'+h)(y'+K)-6(y'+K)^2+7(x'+h)+31(y'+K)-18=0$$

$$x'^2+2hx'+h^2+x'y'+Kx'+hy'+hK-6y'^2-12Ky'-6K^2$$
$$+7x'+7h+31y'+31K-18=0$$

or

$$\boxed{x'^2+x'y'-6y'^2+x'(2h+K+7)+y'(h-12K+31)+(h^2+hK-6K^2+7h+31K-18)=0} \quad (1)$$

to eleminat 1st degree terms, Then

$$\boxed{2h+K+7=0 \quad \& \quad h-12K+31=0} \xrightarrow{\text{solve}} h=\frac{-23}{5}, \quad K=\frac{11}{5}$$

Then the eq.n (1) will be

$$x'^2+x'y'-6y'^2=0$$

Then the eq.n of bisectors will be

$$\frac{x'^2-y'^2}{1-(-6)}=\frac{x'y'}{1/2} \Rightarrow x'^2-y'^2=14x'y'$$

$$x'^2-14x'y'-y'^2=0 \Rightarrow \text{put } x'=\overset{x-h}{x+\frac{23}{5}}, \quad y'=\overset{y'-K}{y-\frac{11}{5}}$$

$$\left(x+\frac{23}{5}\right)^2-14\left(x+\frac{23}{5}\right)\left(y-\frac{11}{5}\right)-\left(y-\frac{11}{5}\right)^2=0$$

$$x^2+\frac{46}{5}x+21.16-14(xy-2.2x+4.6y-10.12)-y^2+4.4y-4.84=0$$

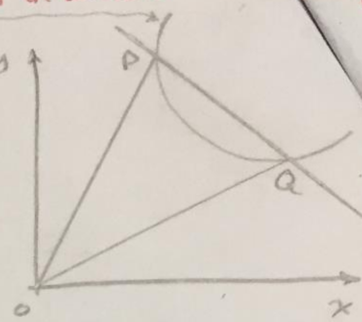
$$\boxed{x^2-14xy-y^2+40x-60y+158=0}$$ * eq.n of bisectors.

Eq.n of the Pair of lines joining the origin to the Points in which

the line $\boxed{lx + my = n}$ $\xrightarrow{}$ PQ eq.n intersects the Conic

$$\boxed{a x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0} \longrightarrow (1)$$

دي معادلة



is

$$\boxed{a x^2 + 2hxy + by^2 + (2gx + 2fy)\left(\frac{lx+my}{n}\right) + c\left(\frac{lx+my}{n}\right)^2 = 0}$$

دي كده معادلة ال line $\overline{OP}$
وال line $\overline{OQ}$ مع بعض

لاحظ

معادلة ال line $\overline{PQ}$
هي ال $lx + my = n$

This is called homogenising → جعل المعادلة ① متجانسة من الدرجة الثانية
لكل الحدود فيها.

Exercises:

1. Show that the lines joining the origin to the intersections of the Conic $7x^2 + 8xy - 7y^2 + 6x - 12y = 0$ and the line $2x + y - 1 = 0$ are at right angles.

Ans. $2x + y = 1$

Homogenising Conic eq.n, Then

$7x^2 + 8xy - 7y^2 + (6x - 12y)(2x + y) = 0,$

$7x^2 + 8xy - 7y^2 + 12x^2 - 18xy - 12y^2 = 0$

hence, $\boxed{19x^2 - 10xy - 19y^2 = 0}$ is the eq.n of the lines

i.e. $a = 19$, $b = -19$

The Condition of right angles Pair of lines is

$a + b = 0$ --> check for our lines

$19 + (-19) = 0$ --> Then

the lines $\boxed{19x^2 - 10xy - 19y^2 = 0}$ are at right angles.

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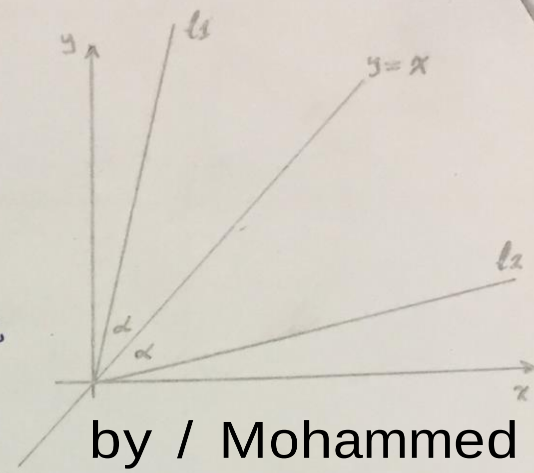
3. Prove that the eq.n to the straight lines through the origin, each of which makes an angle α with the line $y=x$ is

$$x^2 - 2xy\sec 2\alpha + y^2 = 0$$



Ans.

let the eq.n of the required lines is

$$\boxed{a x^2 + 2h\,xy + b y^2 = 0} \rightarrow (1)$$

فرضنا الصورة دي لأنه
قال إنهم بيمروا بال (0,0)

now, the line $y=x$ can be Considered as a bisector,

Then, for eq.n (1), the eq.n of bisectors are

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h} \rightarrow (I)$$

or $\boxed{x^2 - y^2 = \frac{(a-b)}{h} xy} \Rightarrow$ put $x=y \Rightarrow \frac{(a-b)}{h} x^2 = 0$

Then $a-b=0 \Rightarrow \boxed{a=b} \rightarrow$ sub. in (1)

$$\therefore \boxed{a x^2 + 2h\,xy + a y^2 = 0} \rightarrow (2)$$

also,

$\therefore$ $x-y=0$ is one of the bisectors, then $x+y=0$ should be the other one as they are Perpendicular, then $x^2-y^2=0$ is the Pair of bisectors,

Compare $x^2-y^2=0$ with (I) we get $\frac{a-b}{h}=0 \Rightarrow a=b$

also, we have the angle bet.n lines in (2) are

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} \rightarrow \text{put } a=b$$

$$\therefore \tan\theta = \frac{2\sqrt{h^2-a^2}}{2a} \Rightarrow a^2.\tan^2\theta = h^2 - a^2$$

$h^2 = a^2(1+\tan^2\theta)$ --> from Trignometric identities $1+\tan^2\theta = \sec^2\theta$

$\therefore$ $h^2 = a^2\sec^2\theta$ --> here $\theta = 2\alpha$

$\therefore$ $h = \pm a\sec 2\alpha$ --> sub. in (2), Then

$a x^2 \pm 2a\sec 2\alpha \,.\, xy + a y^2 = 0$ --> devide by a

$$\therefore \boxed{x^2 + 2xy\sec 2\alpha + y^2 = 0} \text{ or } \boxed{x^2 - 2xy\sec 2\alpha + y^2 = 0}$$

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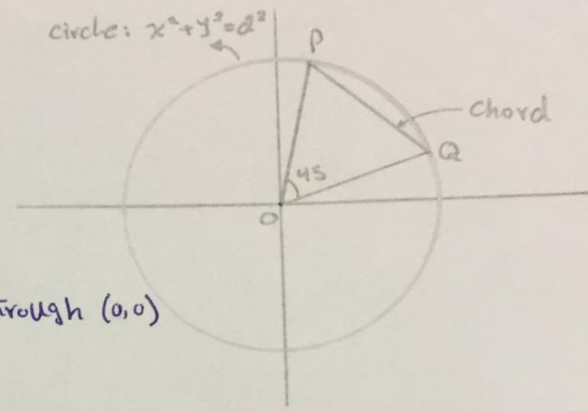
يقابل

2. If $lx + my = 1$ is a chord (وتر) of the circle $x^2 + y^2 = a^2$ which subtends an angle $45°$ at the origin, show that

$$4\left[a^2(l^2+m^2)-1\right] = \left[a^2(l^2+m^2)-2\right]^2.$$



Ans.

The eq.n of the pairs $\overline{OQ}$ & $\overline{OP}$ will take the form.

$$\boxed{Ax^2 + 2H.xy + By^2 = 0} \longrightarrow ①$$ as they pass through $(0,0)$

also, we will obtain them by homogenising $x^2 + y^2 - a^2 = 0$ as follow,

$$x^2 + y^2 - a^2(lx+my)^2 = 0 \Rightarrow x^2 + y^2 - a^2(l^2x^2 + 2lm\,xy + m^2y^2) = 0$$

$$\boxed{(1-a^2l^2)x^2 + (1-a^2m^2)y^2 + (-2a^2lm)xy = 0} \longrightarrow ②$$

Compare ① and ②

$\therefore A = 1-a^2l^2$, $H = -a^2lm$, $B = 1-a^2m^2$, also

The angle $\theta = 45$ have the following formula,

$$\tan\theta = \frac{2\sqrt{H^2-AB}}{A+B} \Rightarrow \underbrace{\tan 45}_{1} = \frac{2\sqrt{a^4l^2m^2-(1-a^2m^2)(1-a^2l^2)}}{2-a^2(l^2+m^2)}$$

$$\therefore 4\left[\cancel{a^4l^2m^2} - 1 + a^2l^2 + a^2m^2 - \cancel{a^4l^2m^2}\right] = \left[2-a^2(l^2+m^2)\right]^2, \text{Then}$$

$$\boxed{4\left[a^2(l^2+m^2)-1\right] = \left[a^2(l^2+m^2)-2\right]^2}$$

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end of ch.2

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